Theor Appl Genet (1986) 71:622–630

Theoretical studies on the necessary number of components in mixtures

2. Number of components and yielding-ability

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Received July 17, 1985; Accepted August 7, 1985
Communicated by P. M. A. Tigerstedt

Summary. Theoretical studies on the necessary number of components in mixtures (for example multiclonal varieties or mixtures of lines) have been performed according to yielding ability. All theoretical investigations are based upon a Gram-Charlier frequency distribution of the component means with skewness γ_1 and kurtosis γ_2. The selected fraction p of the best components constitutes the mixture under consideration. The same selection differential $S = S(p, \gamma_1, \gamma_2)$ can be realized by different parameter values of p, γ_1 and γ_2. Therefore, equal yield levels of the mixture can be achieved by different selected fractions p which implies different numbers of components in the mixture. Numerical results of $S = S(p)$ for different values of γ_1 and γ_2 are presented and discussed. Of particular interest are the selected fractions p which lead to a maximal selection differential S. These results on S for 'large populations' must be reduced in the case of finite population size. For this correction term we used an approximation $B = B(p, n, \gamma_1, \gamma_2)$ given by Burrows (1972) where n = number of selected components. For given parameter values of γ_1, γ_2 and p, the necessary number n of components can be calculated by using the condition: Burrows-correction less than a certain percentage g of S – for example with $g = 0.05$ or $g = 0.01$. For given γ_1 and γ_2, the number n leading to a maximal selection differential S can be regarded as necessary number of components (necessary = maximum gain of selection under the given conditions). Numerical results are given for $\gamma_2 = 0$ and for eight situations which are defined by linear relations $\gamma_2 = c\gamma_1$ between skewness and kurtosis. These cases will contain all possible numerical situations for γ_1 and γ_2, which may be relevant for practical applications. The necessary number of components turns out to be nearly independent of the numerical value of the kurtosis γ_2. The n-intervals leading to selected fractions p from 0.01 to 0.20 approximately are: $2 \leq n \leq 4$ for $g = 0.05$, $6 \leq n \leq 20$ for $g = 0.01$ and $11 \leq n \leq 40$ for $g = 0.005$, respectively. However, percentages g less than 0.01 would be unrealistically excessive. Therefore, following the assumptions and restrictions given in this paper one may conclude that $n = 20$ seems to be an appropriate upper bound for the necessary number of components in mixtures.

Key words: Mixtures – Number of components – Yielding ability

Introduction and problem

In the last few years essential improvements of methods of vegetative propagation have been achieved, for example, by using cuttings and tissue culture techniques. In many fields, such as in forest tree breeding, clones can be produced to such an extent as to be relevant for practical applications. Most forest tree breeders propose the development of multiclonal varieties (= mixtures of different clones artificially created with definite proportions) to maintain some genetic diversity in the stands and to avoid the negative experiences with genetically uniform varieties which are well-known in agricultural crop science.

The models and concepts of the following theoretical investigations have been formulated according to this field of applications. Nevertheless, these studies and results are also of an extended validity to agricultural crop science and plant breeding: 1) including multilines (= mixtures of isolines that differ by single, major genes for reaction to a pathogen) and 2) including mixtures of an arbitrary number of pure lines which are more different among each other than isolines. To provide a simultaneous discussion of these situations – multilines, multiclonal varieties and mixtures of lines – we use the general terms 'mixture' and 'components'.

The main problem confronted in this paper "evaluation of necessary numbers of components in mixtures" has previously been discussed in several investigations but these studies were mainly concerned with special aspects and problems (see Hühn 1985). No general theoretical approach has yet been worked out.

In a previous paper (Hühn 1985) the problem of necessary numbers of components in mixtures was discussed according to yield stability. In the present paper some statistical approaches and numerical results concerning the necessary number of components in mixtures with respect to yielding-ability are presented.

In these investigations we don't consider successive generations. Only one period from the initial composition of the mixture until the final harvest shall be analysed.

An increase in the number of components in mixtures leads to a decrease of the mean yield-level. This often cited argument requires some comment: undoubtedly, the best component will be higher-yielding than each mixture of each number of selected components (assumption: no distinct positive competitive and mixing effects).

More generally: for selected fractions p_1 and p_2 with $p_2 > p_1$ we have $S_2 < S_1$, where S_1 and S_2 are the corresponding selection differentials. This argumentation will be only true with regards to a selection in a definite population with a definite distribution of component means. Nevertheless, the same selected fraction and, consequently, the same selection differential (for equal variability) can be realized with different numbers of selected components if the sizes of the selected populations are different. Selecting n_1 components from a population of size N_1 and selecting n_2 components with $n_2 > n_1$ from a population of size $\frac{n_2}{n_1} N_1$ will result in equal selected fractions and therefore (for equal variabilities in both populations) in equal selection differentials. Assuming an equal yield level of populations with sizes N_1 and $\frac{n_2}{n_1} N_1$ the same yield of a mixture consisting of all selected components can be realized with very different numbers of components.

This simple argument concerning the realization of equal selection differentials using unequal numbers of components can be generalized and precised considerably.

Theoretical investigations and some numerical results

Each mixture shall be composed of equal proportions of it's components. Breeders will predominantly propagate the higher-yielding plants vegetatively. Therefore, the frequency distribution of the component means of the basic population wherein the components of the mixture must be selected will be negatively skewed with skewness γ_1 and $\gamma_1 \neq 0$.

Furthermore, this distribution will show a kurtosis γ_2 with $\gamma_2 \neq 0$. In most situations a negative kurtosis can be expected because common breeding practices will result in distributions which are more deflated and extended with a lower maximum than the normal distribution.

Such frequency distributions with skewness $\gamma_1 \neq 0$ and kurtosis $\gamma_2 \neq 0$ can often be well approximated by the Gram-Charlier-distribution with density-function $f^G(x)$ (Kendall and Stuart 1958):

$$f^G(x) = f^N(x) \cdot \left[1 + \frac{\gamma_1}{6} H_3(z) + \frac{\gamma_2}{24} H_4(z)\right] \qquad (1)$$

where:

x = variable with mean μ and variance σ^2

$z = \frac{x-\mu}{\sigma}$ (standardized variable)

$H_3(z)$ and $H_4(z)$ = Hermite-Tschebycheff-polynomials with $H_3(z) = z^3 - 3z$ and $H_4(z) = z^4 - 6z^2 + 3$

γ_1 = skewness of the distribution of the component means

γ_2 = kurtosis of the distribution of the component means

$f^N(z) = (\sqrt{2\pi})^{-1} \exp(-z^2/2)$ (density of the standardized normal distribution).

All following theoretical investigations are based upon the Gram-Charlier-distribution for the standardized variable z:

$$f^G(z) = f^N(z) \cdot \left[1 + \frac{\gamma_1}{6} H_3(z) + \frac{\gamma_2}{24} H_4(z)\right]. \qquad (2)$$

Selecting a fraction p of the best components gives:

$$\int_h^\infty f^G(z)\,dz = p \qquad (3)$$

with: h = selection point (in standardized form).

The selection differential S (= difference between the mean of the selected components and the mean of the unselected population) can be expressed as:

$$S = \frac{\int_h^\infty z\, f^G(z)\,dz}{p}. \qquad (4)$$

From (3) we obtain:

$$p = \left[R(h) + \frac{\gamma_1}{6}(h^2 - 1) + \frac{\gamma_2}{24}(h^3 - 3h) \right] \cdot f^N(h) \qquad (5)$$

where R (h) denotes Mill's ratio:

$$R(h) = \frac{\int_h^\infty f^N(z)\,dz}{f^N(h)}. \qquad (6)$$

Formula (4) together with (5) gives:

$$S = \frac{1 + \frac{\gamma_1}{6} h^3 + \frac{\gamma_2}{24}(h^4 - 2h^2 - 1)}{R(h) + \frac{\gamma_1}{6}(h^2 - 1) + \frac{\gamma_2}{24}(h^3 - 3h)}. \qquad (7)$$

Intensity of selection can be described by the selection point h or by the selected fraction p. Usually the percentage p will be given. For given γ_1, γ_2 and p the selection point h can be computed from (5). Combining with (7), we therefore can express the selection differential S as a function of γ_1, γ_2 and p:

$$S = S(p, \gamma_1, \gamma_2). \qquad (8)$$

The same selection differential S can be realized by different parameter values of p, γ_1 and γ_2 – for example:

$$\left.\begin{array}{lll} p = 2.9; & \gamma_1 = -1.0; & \gamma_2 = -1.0 \\ p = 5.0; & \gamma_1 = -1.5; & \gamma_2 = 0.5 \\ p = 11.6; & \gamma_1 = -1.5; & \gamma_2 = -1.5 \\ p = 15.3; & \gamma_1 = -1.5; & \gamma_2 = 2.0 \\ p = 24.8; & \gamma_1 = -0.5; & \gamma_2 = 2.0 \\ p = 30.9; & \gamma_1 = 0; & \gamma_2 = 0 \end{array}\right\} \Rightarrow S = 1.14.$$

Dependent on the numerical values of skewness and kurtosis of the distribution of the component means the same selection differential S and therefore the same yield level of a mixture can be realized by very different selected fractions p which implies very different numbers of components in the mixture. This aspect – equal yield level of mixtures with different numbers of components – stresses exactly the main problem and emphasis of this paper.

Extensive numerical calculations of S = S (p) for different values of γ_1 and γ_2 are presented in Hühn (1984).

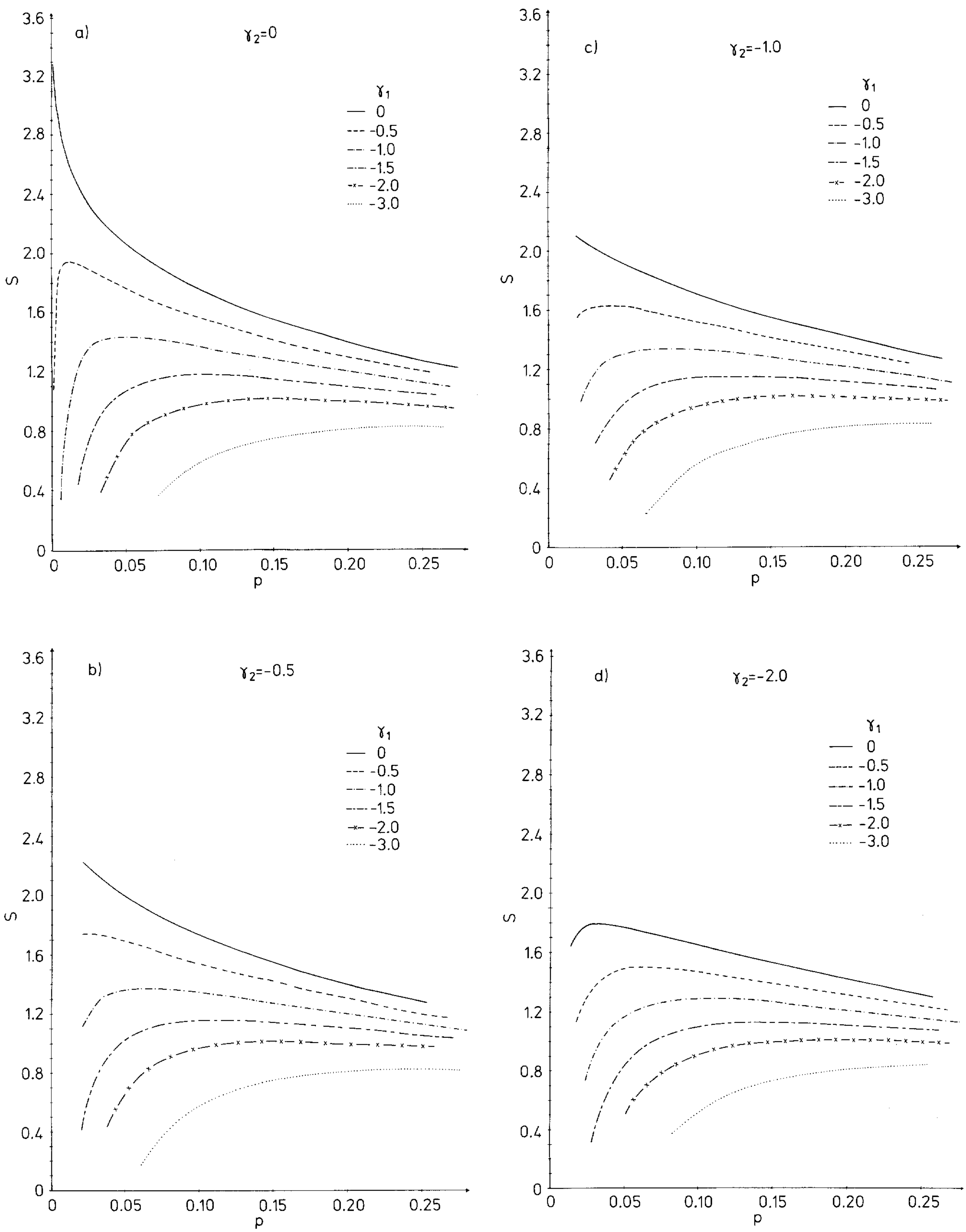
Two situations are of main interest for practical applications: A) $\{\gamma_1 \le 0, \gamma_2 \le 0\}$ and with some lower relevance B) $\{\gamma_1 \le 0, \gamma_2 > 0\}$. Only these two situations shall be discussed here in some detail. To illustrate the numerical results for A) and B) some examples have been selected and demonstrated in Figs. 1 a–d and 2 a–d.

In situations A) and B) the dependence of S on p is hardly influenced by the numerical value of γ_2 (exception: low p-values) (Figs. 1 a–d and 2 a–d). These low percentages p are of no interest in applications. Therefore, the main conclusions shall be only discussed here with regard to $\gamma_2 = 0$ (Fig. 1 a):

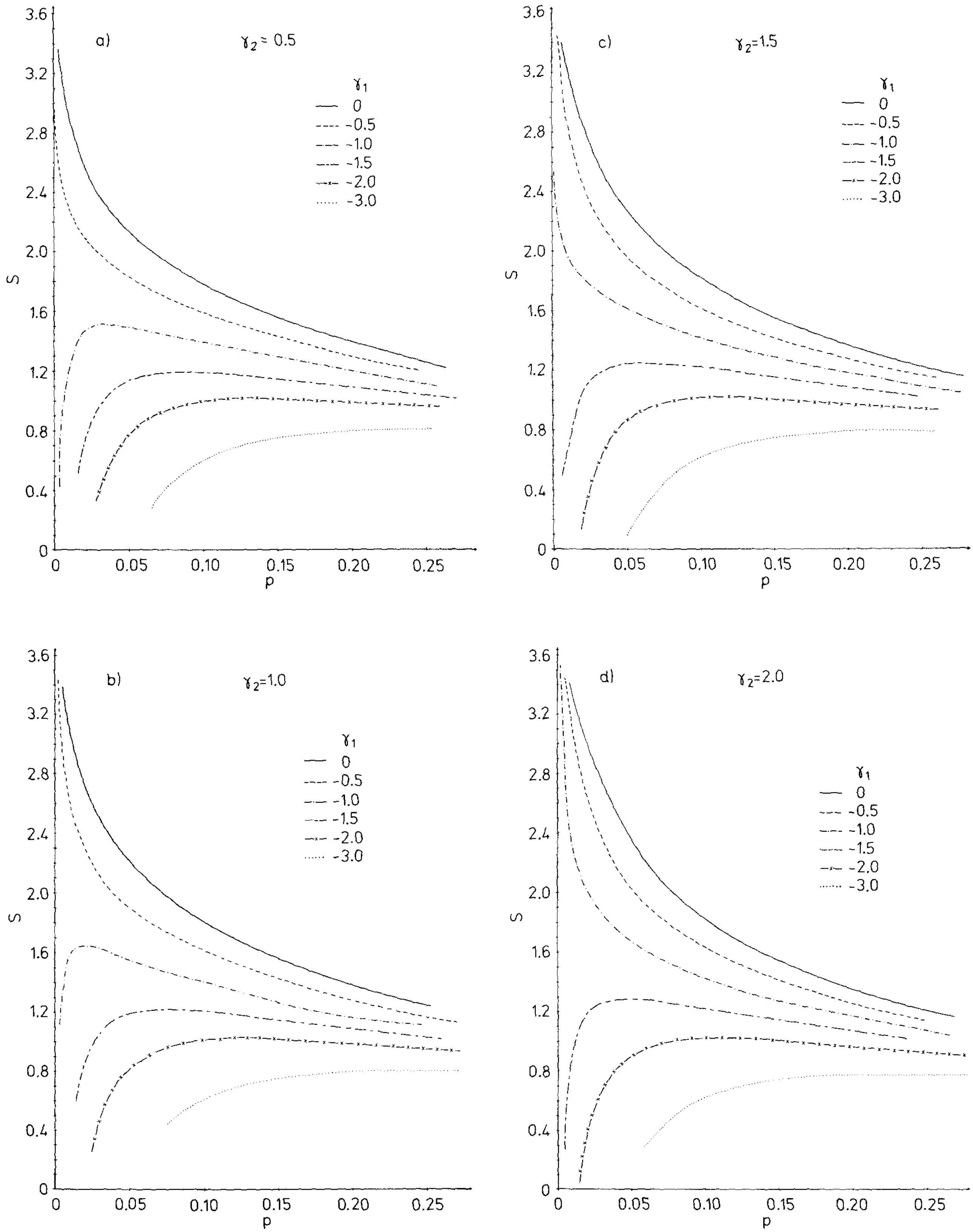
1. S decreases with increasing negative skewness (for each p).
2. All skewed distributions ($\gamma_1 \neq 0$) are characterized by maxima of S = S (p). With increasing negative skewness these maxima are located at increasing p-values. Therefore, different p-values exist leading to equal selection differentials S. The same S and, therefore, the same yield level of a mixture thus can be realized by different numbers of components!
3. Differences between selection differentials for different γ_1 (for the same p) decrease with increasing p.
4. For large negative skewnesses there are extensive p-intervals (in the range of higher p's) wherein the

Table 1. Selected fractions p which lead to a maximal selection differential S if the distribution of the component means is of skewness γ_1 and of no kurtosis ($\gamma_2 = 0$), and the corresponding necessary numbers n for g = 0.05, 0.01 and 0.005

γ_1	p	S	n (for g = 0.05)	n (for g = 0.01)	n (for g = 0.005)
−4.44	39.6	0.65	5	26	51
−3.92	34.6	0.70	5	24	48
−3.48	30.3	0.75	5	23	46
−3.11	26.6	0.80	4	22	43
−2.79	23.4	0.85	4	20	40
−2.52	20.5	0.90	4	19	38
−2.28	18.0	0.95	4	18	35
−2.07	15.9	1.00	4	17	33
−1.88	13.9	1.05	3	16	31
−1.72	12.3	1.10	3	15	29
−1.57	10.8	1.15	3	14	27
−1.44	9.4	1.20	3	13	26
−1.33	8.3	1.25	3	12	24
−1.23	7.3	1.30	3	12	23
−1.13	6.4	1.35	3	11	22
−1.04	5.6	1.40	2	11	21
−0.97	4.9	1.45	2	10	19
−0.91	4.2	1.50	2	9	18
−0.86	3.6	1.55	2	9	17
−0.79	3.2	1.60	2	8	16
−0.73	2.8	1.65	2	8	16
−0.69	2.4	1.70	2	8	15
−0.64	2.1	1.75	2	7	14
−0.60	1.8	1.80	2	7	13
−0.57	1.6	1.85	2	7	13
−0.53	1.4	1.90	2	7	13
−0.50	1.2	1.95	2	6	12
−0.47	1.0	2.00	2	6	11
−0.44	0.9	2.05	2	6	11
−0.42	0.7	2.10	1	5	10
−0.40	0.6	2.15	1	5	10
−0.38	0.5	2.20	1	5	9



Figs. 1 a – d. Selection differential S dependent on selected fraction p for different numerical values of skewness γ_1 and kurtosis γ_2 (case A)



Figs. 2 a – d. Selection differential S dependent on selected fraction p for different numerical values of skewness γ_1 and kurtosis γ_2 (case B)

S-values change only slightly. For example: For $\gamma_1 = -2.0$ the S-values in the p-interval from 7.2% to 34.5% only change from S = 0.90 to S = 1.02.
For further results and discussions see Figs. 1a–d and 2a–d and Hühn (1984).

If we consider the selection differential S as a function of the selection point h, S = S (h), it is known from theory that the maximum of S exists for S = h (James 1976). S = h together with (7) gives:

$$\gamma_1 = \frac{6 \cdot [h \cdot R(h) - 1]}{h} + \frac{\gamma_2}{4} \cdot \frac{1 - h^2}{h}. \qquad (9)$$

For given γ_1 and γ_2 a selection with selection point h computed from (9) leads to a maximal selection differential S. The corresponding selected fraction p can then be computed by (5).

Detailed and extended numerical calculations for different $\gamma_1 \neq 0$ and $\gamma_2 \neq 0$ are presented in Hühn (1984). In the present article only some selected results for the special case $\gamma_2 = 0$ shall be given (left part of Table 1). With decreasing skewness the selection differential which can be realized maximally increases. Therefore, one should apply for a most reduced skewness in the original material wherein the components for the mixture have to be selected.

The foregoing results on selection are valid for 'large populations' (= infinite population size). For finite population size the selection differential S turns out to be somewhat smaller than in the infinite case. Tables of the selection intensity for finite populations are available in the literature (Nanson 1967; Harter 1970; Becker 1975).

For practical applications an approximation given by Burrows (1972) seems to be of sufficient accuracy:

$$\text{Burrows-correction} = B = \frac{N - n}{2N(N+1)f(h)} \qquad (10)$$

where:

N = total number of components
n = number of selected components
f (h) = ordinate of the standardized frequency distribution at selection point h.

In finite populations the selection differential S must be reduced by this amount B. Application to the Gram-Charlier-distribution (2) gives:

$$B = \frac{N - n}{2N(N+1)f^N(h)\left[1 + \frac{\gamma_1}{6} H_3(h) + \frac{\gamma_2}{24} H_4(h)\right]}. \qquad (11)$$

If one introduces the percentage p of selected components (p = n/N) one obtains:

$$B = \frac{1 - p}{2\left(\frac{n}{p} + 1\right) f^N(h)\left[1 + \frac{\gamma_1}{6} H_3(h) + \frac{\gamma_2}{24} H_4(h)\right]}. \qquad (12)$$

For given parameter values of γ_1 and γ_2 and a required selected fraction p the following procedure can be applied to obtain estimates for the necessary number n of components:

1. Calculation of h by (5).
2. Calculation of S by (7).
3. Calculation of n by (12) using the condition: Burrows-correction less than a certain percentage g of S – for example with g = 0.05 or g = 0.01.

If we want to realize a maximal selection differential S (that is: validity of (9)) for given γ_1 and γ_2, the number n of components calculated in line with steps 1–3 from above can be regarded as the necessary number of components (necessary = maximal selection differential under the given conditions). In this situation the computational procedure must be as follows:

1. Simultaneous calculation of h and S (because of the maximum property S = h) by (9).
2. Calculation of p by (5).
3. Calculation of n by (12) using the condition: Burrows-correction less than a certain percentage g of S.

For the numerical values of γ_1, p and S given on the left side of Table 1 (for the special case: $\gamma_2 = 0$), the corresponding numbers n are calculated and presented on the right side of Table 1 (for g = 0.05, 0.01 and 0.005).

With decreasing p, i.e. an increasing selection intensity, n decreases. For practical applications the most interesting interval will be $0.01 \le p \le 0.18$. For this p-interval the necessary numbers n are $11 \le n \le 35$ (for g = 0.005), $6 \le n \le 18$ (for g = 0.01) and $2 \le n \le 4$ (for g = 0.05) (Table 1). These results are valid for $\gamma_2 = 0$.

To provide an analogous study of the general case with $\gamma_2 \neq 0$ numerical calculations of n have been performed using those parameter values leading to a maximal selection differential S (i.e. validity of (9)). To reduce the computational labour we don't start with arbitrary γ_1- and γ_2-values but propose a linear relation $\gamma_2 = c\gamma_1$ (with c = const) between γ_1 and γ_2.

In spite of this simplification the computational work in performing steps 1–3 is time-consuming and troublesome. This will be particularly true for step 1: Calculation of h (for given γ_1 and γ_2) by (9). To facilitate this calculation we used the following approach:

1. For given h and a definite relation $\gamma_2 = c\gamma_1$ between γ_1 and γ_2 one can easily calculate γ_1 by (9) and afterwards γ_2 by $\gamma_2 = c\gamma_1$. S is known by the maximum-condition S = h.
2. Calculation of p by (5).
3. Calculation of n by (12) using the condition: Burrows-correction less than a certain percentage g of S.

This computational procedure can be easily applied and gives 1) selected fractions p which lead to a maxi-

Table 2a. Selected fractions p which lead to a maximal selection differential S if the distribution of the component means is of skewness γ_1 and kurtosis γ_2 together with the corresponding necessary numbers n for g = 0.05, 0.01 and 0.005 (for $\gamma_2 = \pm \frac{1}{2}\gamma_1$ and $\gamma_2 = \pm \gamma_1$)

$\gamma_2 = \pm \frac{1}{2}\gamma_1$				No. n of components			$\gamma_2 = \pm \gamma_1$				No. n of components		
γ_1	γ_2	p	S	(for g = 0.05)	(for g = 0.01)	(for g = 0.005)	γ_1	γ_2	p	S	(for g = 0.05)	(for g = 0.01)	(for g = 0.005)
−5.84	−2.92	54.6	0.6	5	26	51	−6.90	−6.90	67.1	0.6	4	22	43
−3.30	−1.65	30.7	0.8	5	22	45	−3.51	−3.51	35.3	0.8	5	23	46
−2.58	−1.29	23.4	0.9	4	20	39	−2.07	−2.07	20.0	1.0	4	18	36
−2.07	−1.03	17.9	1.0	4	17	35	−1.64	−1.64	15.2	1.1	3	16	32
−1.68	−0.84	13.8	1.1	3	15	30	−1.32	−1.32	11.6	1.2	3	14	28
−1.38	−0.69	10.6	1.2	3	14	27	−1.08	−1.08	8.9	1.3	3	12	25
−1.15	−0.57	8.1	1.3	3	12	24	−0.89	−0.89	6.8	1.4	3	11	22
−0.96	−0.50	6.2	1.4	3	11	21	−0.75	−0.75	5.1	1.5	2	10	20
−0.82	−0.41	4.7	1.5	2	10	19	−0.63	−0.63	3.9	1.6	2	9	18
−0.70	−0.35	3.6	1.6	2	9	17	−0.54	−0.54	2.9	1.7	2	8	16
−0.60	−0.30	2.6	1.7	2	8	15	−0.46	−0.46	2.2	1.8	2	7	14
−0.52	−0.26	2.0	1.8	2	7	14	−0.40	−0.40	1.6	1.9	2	7	13
−0.45	−0.23	1.5	1.9	2	7	13	−0.34	−0.34	1.2	2.0	2	6	12
−0.40	−0.20	1.1	2.0	2	6	12	−0.30	−0.30	0.9	2.1	2	6	11
−0.31	−0.15	0.6	2.2	1	5	10	−0.26	−0.26	0.6	2.2	1	5	10
−7.43	3.72	66.3	0.4	5	28	55	−6.15	6.15	55.5	0.4	6	31	62
−4.47	2.23	38.4	0.6	5	26	52	−4.00	4.00	32.8	0.6	5	25	50
−2.95	1.47	22.9	0.8	4	20	40	−2.80	2.80	19.7	0.8	4	19	38
−2.45	1.23	17.8	0.9	4	18	35	−2.39	2.39	15.2	0.9	4	17	33
−2.07	1.03	13.8	1.0	3	16	31	−2.07	2.07	11.7	1.0	3	14	29
−1.76	0.88	10.7	1.1	3	14	27	−1.80	1.80	9.0	1.1	3	13	25
−1.51	0.76	8.2	1.2	3	12	24	−1.59	1.59	6.8	1.2	3	11	22
−1.31	0.66	6.3	1.3	3	11	22	−1.41	1.41	5.2	1.3	2	10	20
−1.14	0.57	4.8	1.4	2	10	19	−1.26	1.26	3.9	1.4	2	9	18
−1.01	0.51	3.6	1.5	2	9	17	−1.14	1.14	2.9	1.5	2	8	16
−0.89	0.45	2.8	1.6	2	8	16	−1.04	1.04	2.1	1.6	2	7	14
−0.80	0.40	2.1	1.7	2	7	14	−0.95	0.95	1.6	1.7	2	7	13
−0.71	0.36	1.5	1.8	2	7	13	−0.87	0.87	1.1	1.8	2	6	11
−0.64	0.32	1.1	1.9	2	6	12	−0.81	0.81	0.8	1.9	1	5	10
−0.58	0.29	0.8	2.0	2	6	11	−0.75	0.75	0.6	2.0	1	5	9

mal selection differential S if the distribution of the component means is of skewness γ_1 and kurtosis γ_2 and 2) the corresponding necessary numbers n for a given percentage g. Numerical results are presented in Tables 2a, b for g = 0.05, 0.01 and 0.005 and for eight situations which are defined by the following relations between γ_1 and γ_2: $\gamma_2 = \pm \frac{1}{2}\gamma_1$, $\gamma_2 = \pm \gamma_1$, $\gamma_2 = \pm \frac{3}{2}\gamma_1$ and $\gamma_2 = \pm 2\gamma_1$. These cases will contain all possible numerical situations for skewness and kurtosis which may be relevant for practical applications (Hühn 1984).

The main numerical results on the resulting necessary number of components n are summarized in Table 3, where those n-intervals are presented leading to selected fractions p from 0.01 to 0.20 approximately. Additionally, the special case $\gamma_2 = 0$ has been included.

The resulting n-intervals (for any given percentage g) are nearly constant in the different situations. That means: The necessary number of components turns out to be nearly independent on the numerical value of the kurtosis γ_2.

For the condition 'Burrows-correction less than 0.05 of S' an upper bound for the necessary number n of components results from Table 3 to n = 4. For 0.01 this upper bound increases up to n = 20 and for 0.005 a further increase up to n = 40 can be observed. The corresponding n-intervals are $2 \le n \le 4$ (for g = 0.05), $6 \le n \le 20$ (for g = 0.01) and $11 \le n \le 40$ (for g = 0.005) (see "Discussion").

Discussion

We don't consider successive generations. Only one rotation period from the initial composition of the mixture until the final harvest shall be analysed.

The preceding selection calculations are based upon the frequency distribution of the component means with density $f^G(z)$ (in standardized form). These component means are the component yields at the final harvest stage obtained from pure stand trials of the

Table 2b. Selected fractions p which lead to a maximal selection differential S if the distribution of the component means is of skewness γ_1 and kurtosis γ_2 together with the corresponding necessary numbers n for g = 0.05, 0.01 and 0.005 (for $\gamma_2 = \pm \frac{3}{2}\gamma_1$ and $\gamma_2 = \pm 2\gamma_1$)

$\gamma_2 = \pm \frac{3}{2}\gamma_1$				No. n of components			$\gamma_2 = \pm 2\gamma_1$				No. n of components		
γ_1	γ_2	p	S	(for g = 0.05)	(for g = 0.01)	(for g = 0.005)	γ_1	γ_2	p	S	(for g = 0.05)	(for g = 0.01)	(for g = 0.005)
−3.74	−5.62	40.4	0.8	5	23	46	−4.02	−8.03	46.5	0.8	4	22	45
−2.07	−3.10	22.1	1.0	4	19	37	−2.81	−5.63	33.1	0.9	4	21	42
−1.60	−2.40	16.6	1.1	4	16	32	−2.07	−4.13	24.2	1.0	4	19	38
−1.27	−1.90	12.6	1.2	3	14	28	−1.57	−3.14	18.0	1.1	4	17	33
−1.02	−1.53	9.5	1.3	3	13	25	−1.22	−2.44	13.5	1.2	3	15	29
−0.83	−1.24	7.2	1.4	3	11	22	−0.97	−1.94	10.1	1.3	3	13	26
−0.69	−1.03	5.4	1.5	2	10	20	−0.78	−1.55	7.6	1.4	3	12	23
−0.58	−0.86	4.1	1.6	2	9	18	−0.64	−1.28	5.7	1.5	2	10	20
−0.48	−0.73	3.1	1.7	2	8	16	−0.53	−1.06	4.3	1.6	2	9	18
−0.41	−0.62	2.3	1.8	2	7	15	−0.44	−0.88	3.2	1.7	2	8	16
−0.35	−0.53	1.7	1.9	2	7	13	−0.37	−0.74	2.4	1.8	2	8	15
−0.30	−0.45	1.3	2.0	2	6	12	−0.32	−0.63	1.8	1.9	2	7	13
−0.26	−0.39	0.9	2.1	2	6	11	−0.27	−0.54	1.3	2.0	2	6	12
−0.23	−0.34	0.7	2.2	1	5	10	−0.23	−0.46	1.0	2.1	2	6	11
−0.20	−0.30	0.5	2.3	1	5	9	−0.20	−0.40	0.7	2.2	1	5	10
−5.25	7.88	47.8	0.4	6	32	63	−6.92	13.85	72.0	0.2	8	39	78
−3.62	5.42	28.3	0.6	5	24	47	−5.56	11.12	54.9	0.3	7	37	73
−3.08	4.62	21.8	0.7	4	20	40	−4.58	9.16	42.1	0.4	6	31	62
−2.66	3.99	16.7	0.8	4	17	34	−3.85	7.71	32.3	0.5	5	26	52
−2.33	3.50	12.7	0.9	3	15	30	−3.30	6.60	24.6	0.6	5	22	44
−2.07	3.10	9.6	1.0	3	13	26	−2.87	5.75	18.7	0.7	4	18	37
−1.85	2.78	7.2	1.1	3	11	22	−2.54	5.08	14.0	0.8	3	16	31
−1.67	2.51	5.3	1.2	2	10	19	−2.28	4.55	10.4	0.9	3	13	26
−1.53	2.30	3.9	1.3	2	9	17	−2.07	4.13	7.5	1.0	3	11	22
−1.40	2.10	2.8	1.4	2	8	15	−1.90	3.80	5.3	1.1	2	9	18
−1.32	1.97	1.9	1.5	2	7	13	−1.77	3.54	3.6	1.2	2	8	15
−1.24	1.86	1.3	1.6	2	6	11	−1.67	3.34	2.3	1.3	2	6	12
−1.18	1.77	0.8	1.7	1	5	9	−1.59	3.17	1.4	1.4	1	5	10
−1.13	1.70	0.5	1.8	1	4	8	−1.55	3.10	0.6	1.5	1	3	6
−1.10	1.65	0.3	1.9	1	3	6	−1.53	3.07	0.1	1.6	1	1	1

Table 3. Necessary number n of components with g = 0.05, 0.01 and 0.005 for different relations between γ_1 and γ_2 (including $\gamma_2 = 0$) and selected fractions p from 0.01 to 0.20 approximately

Relation between γ_1 and γ_2	p-interval	Necessary no. n of components		
		for g = 0.05	for g = 0.01	for g = 0.005
$\gamma_2 = 0$	1.0−20.5	2−4	6−19	11−38
$\gamma_2 = \frac{1}{2}\gamma_1$	1.1−23.4	2−4	6−20	12−39
$\gamma_2 = \gamma_1$	0.9−20.0	2−4	6−18	11−36
$\gamma_2 = \frac{3}{2}\gamma_1$	0.9−22.1	2−4	6−19	11−37
$\gamma_2 = 2\gamma_1$	1.0−24.2	2−4	6−19	11−38
$\gamma_2 = -\frac{1}{2}\gamma_1$	1.1−22.9	2−4	6−20	12−40
$\gamma_2 = -\gamma_1$	1.1−19.7	2−4	6−19	11−38
$\gamma_2 = -\frac{3}{2}\gamma_1$	1.3−21.8	2−4	6−20	11−40
$\gamma_2 = -2\gamma_1$	1.4−24.6	1−5	5−22	10−44

components. For these values to determine the yield of a mixture consisting of these components several conditions must be valid:

1. No serious competitive and mixing effects between the different components in the mixture.
2. No change in the composition of the mixture.

The approach of calculating necessary numbers n of components, which has been applied in this paper (Burrows-correction less than g% of the selection differential S), seems to be an arbitrary assumption. What are the appropriate numerical values for g?

Furthermore, the same percentage g must be weighted differently with respect to different numerical values of S. Only a cogent argument leading to a definite numerical value for g would dissipate these objections.

There are several possibilities for overcoming these difficulties. An obvious approach would be: Burrows-correction B should be so small that $S-B$ and S doesn't statistically differ significantly. For this procedure the variances of the selection differentials would be necessary. But these variances are only available for the normal distribution (Burrows 1975). Therefore, these results are not applicable to the present study.

Both approaches

1. B less than a certain percentage g of S and
2. No statistically significant difference between $S-B$ and S are based upon different conditions: For 1. the ratio B/S must be sufficiently small while for 2. the ratio B/σ_S (σ_S = standard deviation of S) is decisive. The resulting necessary numbers n of components obtained by using approaches 1 and 2 may be approximately equal under certain conditions. But these considerations wo'nt be followed up further in this paper.

In spite of this uncertainty with respect to the appropriate numerical value of g, one may conclude that percentages g less than 0.01 would be unnecessary and unrealistically excessive. For $g=0.01$ Table 3 gives an upper bound for n of $n=20$. According to the preceding approach and the restrictions and assumptions given in this paper, $n=20$ seems to be an appropriate necessary number of components in mixtures.

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